Motion in a Plane

Question1

In the projectile motion of a particle on a level ground, which of the following remains constant with reference to time and position?

KCET 2025

Options:

A. Average velocity between any two points on the path

B. Horizontal component of velocity

C. Angle between the instantaneous velocity with the horizontal

D. Vertical component of the velocity of the projectile

Answer: B

Solution:

The only quantity that stays unchanged (no acceleration in that direction) is the horizontal component of the velocity. In symbols:

• If the launch speed is u at an angle θ above horizontal,

$v_x = u\cos\theta$

and since there's no horizontal acceleration, v_x remains the same at all times and positions.

So the correct choice is:

Option B: Horizontal component of velocity.

Question2

Two objects are projected at an angle θ° and $(90 - \theta^\circ)$, to the horizontal with the same speed. The ratio of their maximum vertical

heights is

KCET 2022

Options:

A. $\tan\theta : 1$

B. $1 : \tan\theta$

C. $\tan^2\theta : 1$

D. $1 : 1$

Answer: C

Solution:

We know that, maximum vertical height in projectile motion,

$H = \frac{u^2 \sin^2\phi}{2g}$ [where, ϕ is angle of projection.]

$\Rightarrow \quad H \propto \sin^2\phi$

$\Rightarrow \quad \frac{H_1}{H_2} = \frac{\sin^2\phi}{\sin^2\phi_2} = \frac{\sin^2\theta}{\sin^2(90-\theta)}$

[Given, $\phi_1 = \theta$ and $\phi_2 = 90^\circ - \theta$]

$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

$\therefore \quad H_1 : H_2 = \tan^2\theta : 1$

Question3

The maximum range of a gun on horizontal plane is 16 km. If $g = 10\ ms^{-2}$, then muzzle velocity of a shell is

KCET 2021

Options:

A. $160 \mathrm{~ms}^{-1}$

B. $200\sqrt{2} \mathrm{~ms}^{-1}$

C. $400 \mathrm{~ms}^{-1}$

D. $800 \mathrm{~ms}^{-1}$

Answer: C

Solution:

The maximum range R of a projectile on a horizontal plane is given by the formula:

$R = \frac{v^2 \sin 2\theta}{g}$

For the maximum range, θ is 45°, and $\sin 2\theta = \sin 90^\circ = 1$. Hence, the formula simplifies to:

$R = \frac{v^2}{g}$

We can rearrange this equation to solve for the initial muzzle velocity v:

$v^2 = R \cdot g$

$v = \sqrt{R \cdot g}$

Given:

$R = 16 \mathrm{~km} = 16000 \mathrm{~m}$

$g = 10 \mathrm{~m/s}^2$

Substitute these values into the equation:

$v = \sqrt{16000 \cdot 10}$

$v = \sqrt{160000}$

$v = 400 \mathrm{~m/s}$

Therefore, the muzzle velocity of a shell is **Option C:** $400 \mathrm{~m/s}$.

Question4

The trajectory of projectile is

KCET 2021

Options:

A. semicircle

B. an ellipse

C. a parabola always

D. a parabola in the absence of air resistance

Answer: D

Solution:

The trajectory of a projectile is a parabola in the absence of air resistance.In the presence of air resistance, range and maximum height of projectile, both will decrease, hence path of projectile will not remain parabolic.

Question5

For a projectile motion, the angle between the velocity and acceleration is minimum and acute at

KCET 2021

Options:

A. only one point

B. two points

C. three points

D. four points

Answer: A

Solution:

In projectile motion, centre of acceleration is directed towards the centre of earth and at peak point, the angle between velocity and acceleration is 90°. because at peak point the vertical component of velocity is zero. Thus, it is minimum and acute at only one point (i.e. at highest point)

Question6

A particle starts from the origin at $t = 0$ with a velocity of $10\hat{\mathbf{j}}\mathrm{ms}^{-1}$ and move in the x-y plane with a constant acceleration of $(8\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\mathrm{ms}^{-2}$. At an instant when the x-coordinate of the particle is $16\ \mathrm{m}$, y-coordinate of the particle is

KCET 2021

Options:

A. 16 m

B. 28 m

C. 36 m

D. 24 m

Answer: D

Solution:

Given, $\mathbf{u} = 10\hat{\mathbf{j}}\mathrm{ms}^{-1}, \mathbf{a} = 8\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\mathrm{ms}^{-2}$

$x = 16\ \mathrm{m}$

Using equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = 10\hat{\mathbf{j}} \times t + \frac{1}{2}(8\hat{\mathbf{i}} + 2\hat{\mathbf{j}})t^2$$

$$\Rightarrow \quad 16\hat{\mathbf{i}} + y\hat{\mathbf{j}} = 10\hat{\mathbf{j}} + 4t^2\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}$$

Comparing both sides, we get

$$\begin{aligned} & 4t^2 = 16 \\ \Rightarrow \quad & t = 2\ \mathrm{s} \\ \text{and} \quad & y = 10t + t^2 \\ & = 10 \times 2 + (2)^2 \\ & = 24\ \mathrm{m} \end{aligned}$$

Question7

Rain is falling vertically with a speed of $12\ \mathrm{ms}^{-1}$. A woman rides a bicycles with a speed of $12\ \mathrm{ms}^{-1}$ in east to west direction. What is the direction in which she should hold her umbrella?

KCET 2020

Options:

A. 30°, towards east

B. 45°, towards east

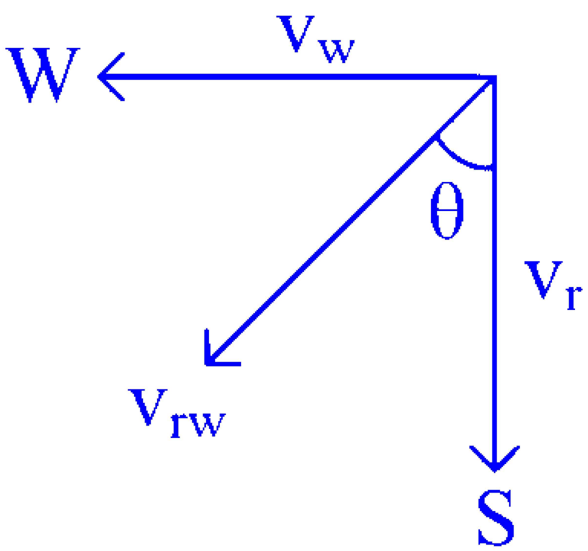
C. 30°, towards west

D. 45°, towards west

Answer: B

Solution:

The given situation is shown below



Here, velocity of rain, $v_r = 12 \text{ ms}^{-1}$

and velocity of woman, $v_w = 12 \text{ ms}^{-1}$

The relative velocity of rain w.r.t. woman is

$$\begin{aligned} v_{nw} &= \sqrt{v_w^2 + v_r^2} \\ &= \sqrt{(12)^2 + (12)^2} = 12\sqrt{2} \text{ ms}^{-1} \end{aligned}$$

The direction in which the woman should hold her umbrella is

$$\sin\theta = \frac{v_w}{v_{rw}} = \frac{12}{12\sqrt{2}}$$
$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ, \text{ towards east}$$

Question8

The trajectory of a projectile projected from origin is given by the equation $y = x - \frac{2x^2}{5}$. The initial velocity of the projectile is

KCET 2019

Options:

A. $\frac{2}{5} \text{ ms}^{-1}$

B. 5 ms^{-1}

C. 25 ms^{-1}

D. $\frac{5}{2} \text{ ms}^{-1}$

Answer: B

Solution:

The trajectory of a projectile launched from the origin is represented by the equation:

$y = x - \frac{2x^2}{5}$

We'll compare this equation with the standard trajectory equation for a projectile:

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$

From this comparison, we find:

$\tan\theta = 1 \Rightarrow \theta = 45^\circ$

This indicates that the angle of projection is 45°.

At the range (R), the height ($y = 0$) because the projectile returns to the horizontal axis:

From the given equation:

$0 = R - \frac{2R^2}{5} 0 = R\left(1 - \frac{2R}{5}\right) \quad \Rightarrow \quad R \neq 0 \therefore \quad 1 - \frac{2R}{5} = 0 \quad \Rightarrow \quad R = \frac{5}{2}$

Here, R is the range of the projectile.

Using the range formula for projectiles:

$R = \frac{u^2\sin(2\times45^\circ)}{g}$

Substitute the known values:

$\frac{u^2}{10} = \frac{5}{2} \quad \left[g = 10\,\mathrm{m/s^2}\right] u^2 = \frac{5}{2} \times 10 \quad \Rightarrow \quad u = 5\,\mathrm{m/s}$

Therefore, the initial velocity u of the projectile is $5\,\mathrm{m/s}$.
